OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 2640

Mechanics 4

Wednesday **25 MAY 2005** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
List of Formulae (MF8)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- Where a numerical value for the acceleration due to gravity is needed, use $9.8\,\mathrm{m\,s^{-2}}$.
- You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 60.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

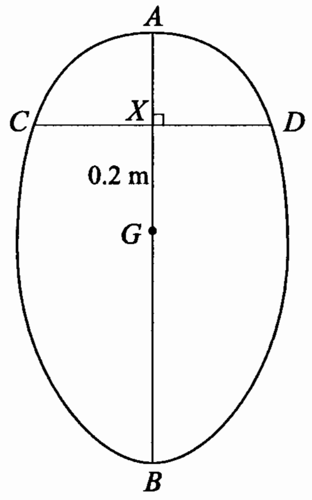
1 A wheel is rotating freely with angular speed $25\,\text{rad s}^{-1}$ about a fixed axis through its centre. The moment of inertia of the wheel about the axis is $0.65\,\text{kg m}^2$. A couple of constant moment is applied to the wheel, and in the next 5 seconds the wheel rotates through 180 radians.

(i) Find the angular acceleration of the wheel. [2]

(ii) Find the moment of the couple about the axis. [2]

2 The region enclosed by the curve $y = \sqrt{x}$ for $0 \leqslant x \leqslant 9$, the x-axis and the line $x = 9$ is occupied by a uniform lamina. Find the coordinates of the centre of mass of this lamina. [7]

3



A lamina has mass 1.5 kg. Two perpendicular lines AB and CD in the lamina intersect at the point X. The centre of mass, G, of the lamina lies on AB, and $XG = 0.2$ m (see diagram). The moment of inertia of the lamina about AB is $0.02\,\text{kg m}^2$, and the moment of inertia of the lamina about CD is $0.12\,\text{kg m}^2$. The lamina is free to rotate in a vertical plane about a fixed horizontal axis perpendicular to the lamina and passing through X.

(i) The lamina makes small oscillations as a compound pendulum. Find the approximate period of these oscillations. [3]

(ii) The lamina starts at rest with G vertically below X. A couple of constant moment 3.2 N m about the axis is now applied to the lamina. Find the angular speed of the lamina when XG is first horizontal. [4]

4 A boat A has constant velocity $12\,\text{m s}^{-1}$ in the direction with bearing 110°. A boat B, which is initially 250 m due south of A, moves with constant speed $6\,\text{m s}^{-1}$ in the direction which takes it as close as possible to A.

(i) Find the bearing of the direction in which B moves. [4]

(ii) Find the shortest distance between A and B in the subsequent motion. [4]

5 In this question, a and k are positive constants.

The region enclosed by the curve $y = ae^{-\frac{x}{a}}$ for $0 \leqslant x \leqslant ka$, the x-axis, the y-axis and the line $x = ka$ is rotated through 2π radians about the x-axis to form a uniform solid of mass m. Show that the moment of inertia of this solid about the x-axis is $\frac{1}{4}ma^2(1 + e^{-2k})$. [8]

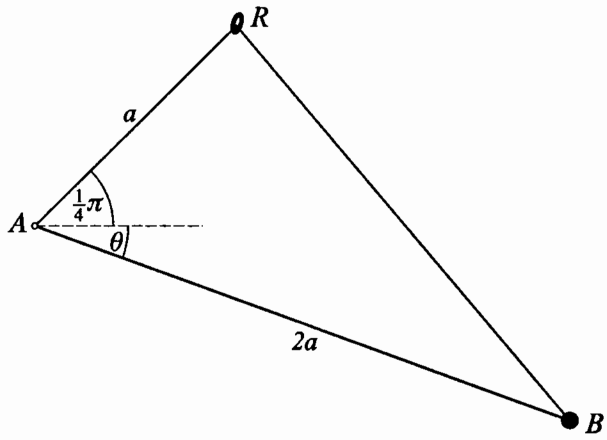
6 A uniform circular disc, of mass m and radius a, has centre C. The disc can rotate freely in a vertical plane about a fixed horizontal axis through the point A on the disc, where $CA = \frac{1}{2}a$. The disc is released from rest in the position with CA horizontal. When the disc has rotated through an angle θ,

(i) show that the angular acceleration of the disc is $\dfrac{2g\cos\theta}{3a}$, [4]

(ii) find the angular speed of the disc, [3]

(iii) find the components, parallel and perpendicular to CA, of the force acting on the disc at the axis. [6]

7

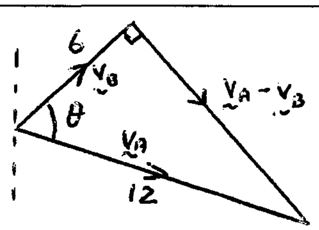


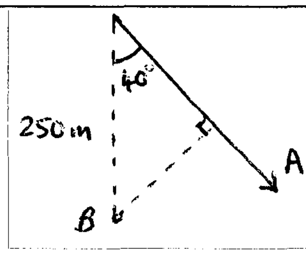
A light rod AB of length $2a$ can rotate freely in a vertical plane about a fixed horizontal axis through A. A particle of mass m is attached to the rod at B. A fixed smooth ring R lies in the same vertical plane as the rod, where $AR = a$ and AR makes an angle $\frac{1}{4}\pi$ above the horizontal. A light elastic string, of natural length a and modulus of elasticity $mg\sqrt{2}$, passes through the ring R; one end is fixed to A and the other end is fixed to B. The rod makes an angle θ below the horizontal, where $-\frac{1}{4}\pi < \theta < \frac{3}{4}\pi$ (see diagram).

(i) Use the cosine rule to show that $RB^2 = a^2\left(5 - (2\sqrt{2})\cos\theta + (2\sqrt{2})\sin\theta\right)$. [2]

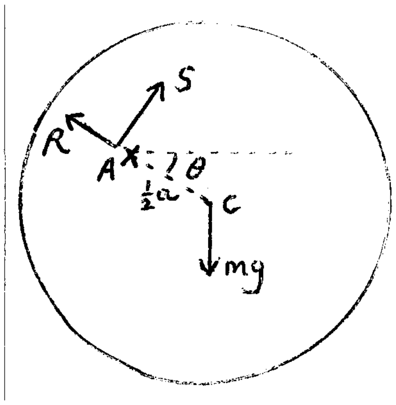
(ii) Show that $\theta = 0$ is a position of stable equilibrium. [6]

(iii) Show that $\dfrac{d^2\theta}{dt^2} = -k\sin\theta$, expressing the constant k in terms of a and g, and hence write down the approximate period of small oscillations about the equilibrium position $\theta = 0$. [5]

1 (i)	$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$, $180 = 25 \times 5 + \frac{1}{2}\alpha \times 25$ $\alpha = 4.4 \text{ rad s}^{-2}$	M1 A1 **2**	
(ii)	Moment $= I\alpha = 0.65 \times 4.4$ $= 2.86 \text{ N m}$	M1 A1 ft **2**	
2	Area $= \int_0^9 \sqrt{x}\,dx = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^9 = 18$ $\int xy\,dx = \int_0^9 x^{\frac{3}{2}}\,dx = \left[\frac{2}{5}x^{\frac{5}{2}}\right]_0^9 = 97.2$ $\bar{x} = \frac{97.2}{18}$ $= \frac{27}{5} = 5.4$ $\int \frac{1}{2}y^2\,dx = \int_0^9 \frac{1}{2}x\,dx = \left[\frac{1}{4}x^2\right]_0^9 = 20.25$ $\bar{y} = \frac{20.25}{18}$ $= \frac{9}{8} = 1.125$	B1 B1 M1 A1 B1 M1 A1 **7**	For $\frac{2}{5}x^{\frac{5}{2}}$ For $\frac{1}{4}x^2$ (*or* $\frac{9}{2}y^2 - \frac{1}{4}y^4$)
3 (i)	$I = 0.02 + 0.12 = 0.14$ Period is $2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{0.14}{1.5 \times 9.8 \times 0.2}}$ $= 1.37 \text{ s}$	B1 M1 A1 **3**	
(ii)	WD by couple is $3.2 \times \frac{1}{2}\pi$ $3.2 \times \frac{1}{2}\pi = 1.5 \times 9.8 \times 0.2 + \frac{1}{2}(0.14)\omega^2$ $\omega = 5.46 \text{ rad s}^{-1}$	B1 M1 A1 ft A1 **4**	For WD = PE + KE
4 (i)	 $\cos\theta = \frac{6}{12}$ $\theta = 60°$ Bearing of *B*'s velocity is $110 - 60 = 050°$	M1 A1 M1 A1 **4**	Relative velocity perpendicular to $\mathbf{v}_B$ Correct velocity triangle

(ii)	As viewed from B: 	M1 A1	Considering relative displacement Relative velocity on bearing 140°
	Shortest distance is $250\sin 40$ $= 161$ m	M1 A1 **4**	

5	$m = \rho\int \pi y^2\,dx = \rho\pi\int_0^{ka} a^2 e^{-\frac{2x}{a}}\,dx$	M1	Integral of $\left(e^{-\frac{x}{a}}\right)^2$ (when finding mass or volume)
	$= \rho\pi\left[-\tfrac{1}{2}a^3 e^{-\frac{2x}{a}}\right]_0^{ka}$	A1	For $\int e^{-\frac{2x}{a}}\,dx = -\tfrac{1}{2}ae^{-\frac{2x}{a}}$
	$= \tfrac{1}{2}\rho\pi a^3(1 - e^{-2k})$	A1	For mass or volume
	$I = \int \tfrac{1}{2}\rho\pi y^4\,dx$ $= \tfrac{1}{2}\rho\pi\int_0^{ka} a^4 e^{-\frac{4x}{a}}\,dx$	M1 A1 ft	Integral of y^4 Correct integral expression (in terms of x)
	$= \tfrac{1}{2}\rho\pi\left[-\tfrac{1}{4}a^5 e^{-\frac{4x}{a}}\right]_0^{ka} = \tfrac{1}{8}\rho\pi a^5(1 - e^{-4k})$	A1	
	$= \dfrac{\tfrac{1}{4}ma^2(1 - e^{-4k})}{1 - e^{-2k}}$	M1	*Dependent on previous M1M1*
	$= \dfrac{\tfrac{1}{4}ma^2(1 - e^{-2k})(1 + e^{-2k})}{1 - e^{-2k}} = \tfrac{1}{4}ma^2(1 + e^{-2k})$	A1 (ag) **8**	*Intermediate step not required, provided no wrong working seen*

6 (i)			
	$I = \frac{1}{2}ma^2 + m(\frac{1}{2}a)^2$	M1	Using parallel axes rule
	$= \frac{3}{4}ma^2$	A1	
	$mg(\frac{1}{2}a\cos\theta) = I\alpha = (\frac{3}{4}ma^2)\alpha$	M1	Or differentiating the energy equation
	$\alpha = \dfrac{2g\cos\theta}{3a}$	A1 (ag) **4**	
(ii)	$\frac{1}{2}I\omega^2 = mg(\frac{1}{2}a\sin\theta)$	M1 A1	Using $\frac{1}{2}I\omega^2$
	$\omega = \sqrt{\dfrac{4g\sin\theta}{3a}}$	A1 **3**	
(iii)		B1	For radial acc'n of *C* is $(\frac{1}{2}a)\omega^2$
	$R - mg\sin\theta = m(\frac{1}{2}a)\omega^2$	M1	$\pm R \pm mg\sin\theta = mr\omega^2$ *or* $kma\omega^2$ (with numerical *k*)
	$R = \frac{5}{3}mg\sin\theta$	A1	
		B1	For transverse acc'n of *C* is $(\frac{1}{2}a)\alpha$
	$mg\cos\theta - S = m(\frac{1}{2}a)\alpha$	M1	*as above*
	$S = \frac{2}{3}mg\cos\theta$	A1 **6**	Direction must be clear *Equations involving horizontal and vertical components can earn B1M1B1M1*
	OR $S(\frac{1}{2}a) = I_G\alpha$ M1		Must use I_G
	$S(\frac{1}{2}a) = (\frac{1}{2}ma^2)\alpha$ A1		
	$S = \frac{2}{3}mg\cos\theta$ A1		

7 (i)	$RB^2 = a^2 + (2a)^2 - 2(a)(2a)\cos(\theta + \frac{1}{4}\pi)$ $= 5a^2 - 4a^2(\cos\theta\cos\frac{1}{4}\pi - \sin\theta\sin\frac{1}{4}\pi)$ $= a^2(5 - 2\sqrt{2}\cos\theta + 2\sqrt{2}\sin\theta)$	M1 A1 (ag) **2**	
(ii)	$V = -mg(2a\sin\theta) + \frac{mg\sqrt{2}}{2a} \times RB^2$ $= \frac{5}{2}\sqrt{2}mga - 2mga\cos\theta$ $\frac{dV}{d\theta} = 2mga\sin\theta$ When $\theta = 0$, $\frac{dV}{d\theta} = 0$, hence equilibrium $\frac{d^2V}{d\theta^2} = 2mga\cos\theta$ When $\theta = 0$, $\frac{d^2V}{d\theta^2} = 2mga > 0$, hence stable	M1 A1 M1 A1 M1 A1 **6**	Considering PE and EE Correctly shown or other method for max / min Correctly shown
(iii)	KE is $\frac{1}{2}m(2a\dot{\theta})^2$ $\frac{5}{2}\sqrt{2}mga - 2mga\cos\theta + 2ma^2\dot{\theta}^2 = E$ Differentiating w.r.t. t, $2mga\sin\theta\dot{\theta} + 4ma^2\dot{\theta}\ddot{\theta} = 0$ $\ddot{\theta} = -\frac{g}{2a}\sin\theta$	B1 M1 M1 A1	 Requires fully correct working
	OR $(mg\cos\theta - T\sin\phi)(2a) = I\ddot{\theta}$, where $T = \frac{mg\sqrt{2}(RB)}{a}$ and $\frac{\sin\phi}{a} = \frac{\sin(\theta + \frac{1}{4}\pi)}{RB}$ M2 $\ddot{\theta} = -\frac{g}{2a}\sin\theta$ A2		or $mg\cos\theta - T\sin\phi = m(2a\ddot{\theta})$ Give A1 if just one minor error
	Period is $2\pi\sqrt{\frac{2a}{g}}$	B1 ft **5**	ft provided that k is in terms of a and g only